Voronoi tessellation in shaping the architectural form from flat rod structure

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Abstract: Shaping original rod structures goes less and less frequently according the traditional methods of design, whereas optimization processes rise in significance. Bionic explorations impact the development in contemporary architecture trends that inspire architects to create more tectonically fascinating solutions. The logic behind forming such structures is effectiveness in the use of materials and energy. The dynamic development of digital technology provides new tools for authors to design increasingly complex spatial forms, often arising as a result of multi-criteria optimization processes. In the times of universal computerization, generative design methods are modern, creative tools of the architects that allow them to create diverse, multi-variant structures based on similar assumptions about the common boundary in the search of synergistic solutions for a new generation in architecture and design. The search for optimal solutions was adopted to study the bionic trends in regular and irregular flat rod structures on the Voronoi-based divisions.

1. Introduction

The inspiration with bionics in architecture is now one of the most interesting trends, the effect of which is the search for eco-effective engineering solutions. To follow nature’s technology, forms shaped in morphogenetic processes have characteristics of natural systems such as: ease of adaptation to the environment and high functional, power and material performance. Consequently, through 3D-modeling computer programs based on algorithms it is possible to generate bionic structures, which are of great interest not only among architects, but also among the engineers responsible for the technical side of architecture. One of the trends which is used more and more often in structural surface digitization in architecture is the Voronoi Diagram.

The Voronoi cells describe many structures found in nature and are visible among others on the wings of a dragonfly, on the carapace of a turtle or in the honeycomb structure (Fig. 1). The Voronoi diagram (Dirichlet tessellation, Voronoi tessellation) is a graph composed of centers (seeds) i.e. edges of a diagram which form a Voronoi

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cell. A plane divided into areas using half lines and segments constitutes Voronoi cells, which are convex polygons. A Voronoi node is the equidistant point from at least three centers (Pazderski, 2012).

The individual Voronoi areas are defined for each center (the point on the plane) as a set of points on the plane which are closest to the relevant center, i.e. for a given set of n points, the plane is divided into n areas in such a way that each point in any cell is closer to a specific point of a set of n points, than the other n-1 points.

Voronoi diagrams are examples of a tessellation discretization of a surface with corresponding nodal points, which make it possible to create an optimal grid constructed with nodal points. The Voronoi tessellation is used for searching for the nearest neighborhood, determining the position function (centers), finding the largest empty circle (cells) and path planning (edges) (Janczyk, 2014). In addition, Dirichlet tessellation is increasingly applied in biology, geophysics, or anthropology (Tarczewski, 2011).

Voronoi diagrams can be used for discretization of the surface as well as space. Inspiration with these structures results in development of two- and three-dimensional structural elements that can be used in computer graphics, architectural design and urban planning.

Fig. 1. The occurrence of Voronoi diagrams in nature – Voronoi cells visible on the dragonfly wing.

2. Application of Voronoi tessellation in shaping rod structures as a leading trend in architecture

Shaping architectural elements and structural forms using the Voronoi Diagram is one of the most important, leading trends in architectural design that seeks new forms of expression.
An interesting example of shaping architectural surface can be seen on the example of the WestendGate canopy in Frankfurt am Main in Germany (Fig. 2). The Just Burgeff architekten and a3lab (Asterios Agkathidis Architecture) project is an example of implementing the Voronoi diagrams in a pedestrian passage canopies with an arboreal support. The project completed in 2010 is part of the renovation of the high-rise Marriott Hotel with hotel rooms and offices. As a result of the entrance transfer to an underground garage, the public space around the skyscraper has been changed. Consequently, the gained space has been planned for recreation purposes. The closeness of neighboring green areas required a project that would convert it into a municipal recreation area. As a result, it was decided to provide the city with an open space and construct a canopy that would serve as a new gateway to the WestendGate. The roof design is based on biomorphic shapes that describe the algorithm of natural growth and the formation of living organisms. The structural grid was optimized by finite-element method (FEM) in order to obtain efficiency by using specific construction material, i.e. steel in this project. The curved canopy surface was tested in specialized computer programs outlining the distribution of forces in the construction of the object. A united form and structure has been acquired as a result of the study. It is important to use computer programs to generate and manage 3d models due to the optimization opportunities and the use of modern production methods. The application of the arboreal support (tree branch-like shape) makes it possible to minimize the number of supports and gives an optimal support of a 1000 m² roof area with a variable height reaching up to 14 meters. The structure was made of steel pipes of the same diameter, but three different types of wall thickness, designated as a result of the structural optimization. A swift installation of welded construction on the site was feasible due to the joint application. The curved canopy was filled with transparent, pneumatic ETFE foil cushions matching the shapes of the individual Voronoi cells.

Voronoi diagrams can also be used in the discretization of spatial forms, the example of which is a pavilion, part of a Grotto project for meditation, made within a research project by the group METALAB Architecture + Fabrication, in collaboration with Gerald D. Hines, Mrs. Jane Blaffer Owen, professors Andrew Vrana, Joe Meppelink, Ben Nicholson and a group of students from the College of Architecture in Houston, Texas (Fig. 2). It was supposed to serve as a permanent landmark on the campus of the University of Houston. The project maps the biomorphic processes and its implementation was possible through the use of digital technology for 3D modeling, 3D scanning and the production of particular elements in the process of CNC Fabrication (Computer Numerical Control) using laser cut stainless steel. The divisions were generated parametrically as minimal surfaces, inspired by forms common in marine organisms. The structural grid is made of stainless steel. The steel frame is connected to the concrete foundation.

The algorithm for forming Voronoi cells inspires architects to create new architectural concepts based on three-dimensional elements combined to make structural forms e.g. The Vertical Village concept based on the project by Yushang Zhang, Rajiv Sewtahal, Riemen Postma and Qianqian Cai architects (Fig. 2). Thanks to the application of geometric solutions in the form of Voronoi diagrams the building is characterized by increased efficiency, and the concept reflects architects’ advanced aspirations in accordance with sustainable development. Residential units consist of
Voronoi cells, and as a result all rooms are generated according to this algorithm, leading to the creation of the '3d plot' or 'Vertical Village' projects.

Fig. 2. (a) The canopy of WestendGate in Frankfurt am Main, using the Voronoi diagrams for discretization of the surface. Source: (WestendGate, 2011); (b) A spatial form of New Harmony pavilion Grotto on campus at the University of Houston, Texas, based on the algorithm of shaping the Voronoi diagrams. Source: (Minner, 2011); (c) The visualization of architectural Vertical Village concept, whose main idea is a structural form based on spatial Voronoi cells. Source: (Furuto, 2011).

3. Research

In the era of universal computerization, the use of the Voronoi diagram in shaping structures is a new and creative working tool for architects, which can be used to model a variety of multivariate solutions. Currently, most computer programs for 3D modeling have the ability to generate structures based on the designated boundary parameters. A skillful use of the scripting language allows the architect to create individual tools that shape optimal architectural forms. Topological exploration of a structure’s surface is particularly interesting from the architectural and constructional point of view, dividing the surface into rod elements. The authors of the study performed analytical tests using the Voronoi divisions to optimize the planar rod structures considering the minimal material consumption criterion.

The study adopted the basic assumptions about: geometric forms, support schemes, and type of load (Fig. 3a). A structural construction model was adopted, which consists of a vertical, self-supporting construction made of steel rods.

The study was based on two metrics: (C) and (D), which are the most efficient solutions for typical, regular arrangements applicable in the construction industry i.e. orthogonal, diagonal, and braced grids (Gawell, 2013). The conducted analyses concerned building different Voronoi rod configurations that relate to the given metrics. The analyses were conducted on digital models. The structure of bionic divisions was generated using an algorithm in the Grasshopper (Rhinoceros 3D), the computational models and strength analysis were done in Autodesk Robot Structural Analysis.

For practical reasons buckling lengths of the researched structure’s elements were applied according to truss rods parameters. The rod structure grid was based on two articulated supports. The following fixed and variable loads were applied: the dead load of the construction, the slab floor reactions were treated as a separate variant
included into the combined load with a factor of 1.5 for the variable load, the load of the horizontal design was applied to the upper belt nodes, the load combination was accepted according to the PN-EN 1990.

The section type was the premise for each analyzed system, making them the most effective ones in terms of their load-bearing capacity. Square section S235 profiles were adopted for the analysis. The same cross section was adopted for all tested rod structures. To acquire a fair comparability of the results, the elements were limited to the square pipe sections, RK type, according to the PN-EN 10210-2:2000.

An algorithm was created for the purpose of generating a Voronoi tessellation process in the transformation topology. Preserving the nodes in the advisable boundary points was the first criterion of this process (Fig. 3b, 3c), the second, a presumed, unchangeable field quantity. The generated Voronoi flat structures are a collection of randomly chosen systems indicated by the metrics data algorithm, therefore, the result of the analysis will be the average value that belongs to a given group of structures.

3.1. Analysis 1

The C metric Voronoi structures are the first group of the tested systems (Fig. 4). The number of variants of topological fields and nodes are constant parameters.

The most effective variant of the Voronoi structure was selected as a result of choosing the appropriate section via optimization reasons. The best configuration in view of the minimal load turned out to be the C-V03 variant. The topological transformation of structures in Analysis 1 had a moderate impact on the change in total mass, which amounts to 478.00 kg. The average weight for the structure in the Voronoi C metric is 3528.20 kg. In comparison, the total weight of an orthographic

![Fig. 3. (a) Research design; (b, c) Metrics to search for Voronoi geometry.](image-url)
system for C metric is \(3265.00\) kg.

Tab. 1. Topological structures statement of Analysis 1 and the total mass after optimization.

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of nodes</th>
<th>Number of fields</th>
<th>Profile /type, size./ [mm]</th>
<th>Length of the elements [m]</th>
<th>Weight [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-V01</td>
<td>29</td>
<td>16</td>
<td>RK 220x220x6</td>
<td>88.04</td>
<td>3526</td>
</tr>
<tr>
<td>C-V02</td>
<td>29</td>
<td>16</td>
<td>RK 220x220x6</td>
<td>89.52</td>
<td>3585</td>
</tr>
<tr>
<td>C-V03</td>
<td>29</td>
<td>16</td>
<td>RK 200x200x6</td>
<td>89.28</td>
<td>3239</td>
</tr>
<tr>
<td>C-V04</td>
<td>29</td>
<td>16</td>
<td>RK 220x220x6</td>
<td>89.24</td>
<td>3574</td>
</tr>
<tr>
<td>C-V05</td>
<td>29</td>
<td>16</td>
<td>RK 220x220x6</td>
<td>92.80</td>
<td>3717</td>
</tr>
</tbody>
</table>

3.2. Analysis 2

Analogous Voronoi structures generated for the D metric constitute the second group of systems that take the corresponding topological invariants into account.

Topological transformations for the Voronoi structures in metric D resulted in differences in weight of up to 512.00 kg between the variants. The average mass

![Metric C: V - Voronoi Structure](image)

Fig. 4. Randomly generated topological Voronoi structure variants for the C metric.
for systems in Analysis 2 is 3389.20 kg. Changing the metrics did not improve the quality of the Voronoi structures due to the adopted weight criterion. The total weight is over 3 tons in all variants. The lightest configuration turned out to be the D-V05 structure. In comparison, the total weight of an orthographic system for metric C is 3282.00 kg.

Tab. 2. Topological structures statement of Analysis 2 and the total mass after optimization.

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of nodes</th>
<th>Number of fields</th>
<th>Profile /type, size./ [mm]</th>
<th>Length of the elements [m]</th>
<th>Weight [kg]</th>
</tr>
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<tr>
<td>D-V01</td>
<td>52</td>
<td>25</td>
<td>RK 200x200x6</td>
<td>100.60</td>
<td>3650</td>
</tr>
<tr>
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<td>52</td>
<td>25</td>
<td>RK 200x200x6</td>
<td>101.24</td>
<td>3673</td>
</tr>
<tr>
<td>D-V03</td>
<td>52</td>
<td>25</td>
<td>RK 200x200x5</td>
<td>106.16</td>
<td>3226</td>
</tr>
<tr>
<td>D-V04</td>
<td>52</td>
<td>25</td>
<td>RK 200x200x5</td>
<td>106.48</td>
<td>3236</td>
</tr>
<tr>
<td>D-V05</td>
<td>52</td>
<td>25</td>
<td>RK 200x200x5</td>
<td>104.00</td>
<td>3161</td>
</tr>
</tbody>
</table>

To indicate the desired optimum and the relation between the particular metrics, the least-squares method was applied (Guter and Owćinski, 1967). Graphs of indi-

Fig. 5. Randomly generated topological Voronoi structure variants for the D metric.
Individual analyses were obtained as a result of a third-degree polynomial approximation. As the examples of the Voronoi structures were generated randomly, the graph of the $p^3(x)$ function is variable. For Analysis 1 of Voronoi configuration the desired optimum is between the 1st and 4th topology variant (C-V01 ÷ C-V04). The optimal solutions for Analysis 2 structure are between the 3rd and 5th topology variant (D-V03 ÷ D-V05).

The arrangement of approximate charts used for the analyzed topological variations does not indicate a clear similarity of each surveyed Voronoi group. Regardless of the metric, the structures show a similar constructional efficiency due to the minimum mass. The influence of topological deformation on the Voronoi systems caused mass differences equal to **556.00** kg, and the average total mass for the analyzed Voronoi structures equaled **3458.70** kg.

Unlike the orthogonal, diagonal or diagonally braced structures, the Voronoi partitions can be generated in multivariate systems in compliance with the conditions specified by the designer. The possibility to indicate parameters arising from the functional, aesthetic etc. premises shows greater flexibility in the architectural design. We then have to deal with the architectural and structural efficiency that embodies not one, but many efficient solutions.

![Graph](image)

Fig. 6. Arrangement of the trend line for each topological Voronoi structure analyzed for metrics C and D.

### 4. Summary

The dynamic development of digital technologies provides architects with new tools to shape the increasingly interesting spatial forms as a result of multi-criteria optimization processes. In modeling complex linked systems that accompany the architectural design, algorithms play an important role. Thanks to the algorithm implementation
in the form of a computer program, it is possible to generate complex bionic structures. Within the boundaries of the chosen method generating the shape, it is possible to model an infinite number of given premises, links, etc. and verify the quality of the obtained solutions.

A digital optimization process is an important element in shaping the modern spatial forms inspired by the morphogenesis process of the Voronoi structures. As a result of the study, it was found that the Voronoi structures show the desired efficiency in material consumption, which in comparison to the ability to generate an infinite number of similar solutions suggests that digital Voronoi structure generators have become a vital tool in optimal shaping of systems, often leading to effective, multivariate solutions. The process of area discretization using the Voronoi tessellation can and should be conducted differently in each case, just as there are different user needs and designer ideas.

References


